steady; the casing wall of the malfunctioning fuel assembly is only partially in the molten state here, and the casing wall of the neighboring fuel assembly remains undamaged. Calculation for the case of runoff of molten wall material from the contact zone shows that the internal wall melts somewhat more rapidly: in 4.6 sec from the moment of contact. However, in both cases, if normal heat extraction from the casing wall of the neighboring fuel assembly is maintained, this wall remains undamaged.

## NOTATION

x, coordinate; t, time; T, temperature; $\lambda$, thermal conductivity; c, specific heat of unit volume; $a$, thermal diffusivity; $R_m$, latent heat of fusion; $T_m$, melting point; $\delta$, thickness of casing wall; $X_1$, coordinate of the molten-layer boundary; y, coordinate of the phase boundary; $q_{s1}$, heat flux to the internal (left-hand) boundary; $q_{s2}$, heat flux from the external (right-hand) boundary; $\alpha_2$, heat-transfer coefficient from the external boundary; $t_s$, time of melting of wall; $q_{VF}$, heat liberation in the fuel layer; $\tau$, time step; h, spatial step of the grid. Indices: $\ell$, liquid phase; s, solid phase; i, number of the spatial grid point; j, number of the time step; $N_{y1}$, $N_{y2}$, numbers of the spatial grid points at the phase interface in the j-th and (j + 1)-th steps, respectively; N, number of spatial grid points; ky, number of iteration.

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# TWO-MODE MODEL OF FLOW IN A PLASMOTRON CHANNEL

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The characteristics of an electric arc in a turbulent gas flow are calculated on the basis of the concept of laminar flow in the arc zone.

The methods of calculating electrical arcs in a cylindrical channel which are known in the literature are usually based on the assumption that the flow conditions, which depend on the parameters of the external gas flow blown through the arc, are the same (either laminar or turbulent) over the whole channel cross section. At the same time, taking account of the specific properties of the electric arc allows the flow in the plasmotron channel to be represented in the form of central laminar flow and outer turbulent flow in many cases.

Estimates for various gases show that, at moderate Re (up to $10^5$), calculated from the input parameters, and at sufficiently high temperatures in the central region of the flow ($\sim$15,000 K or more), the mean turbulent thermal conductivity over the channel cross section is approximately an order of magnitude lower than the molecular thermal conductivity, while the corresponding viscosity values are comparable with one another. Similar results were obtained in [1], where it was indicated that the heat transfer in the axial of a plasmotron channel may be regarded as laminar.

Note also the possibility of decrease in the temperature pulsations in a plasma arc on account of radiant heat transfer between turbulent eddies and rapid "deexcitation" of the highest-temperature formations [2]. The result is that turbulent heat transfer is negligibly small, and there is practically no pulsational component of the heat flux for optically dense media with very intense radiation.

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Intense heat liberation and electrical or magnetic fields have an additional laminarizing influence on the flow in the arc column. The action of all these factors may lead to a situation with predominantly laminar flow in the central arc layer and turbulent flow outside that (disregarding the laminar sublayer close to the wall) in the case of high temperatures (the largest currents) and moderate Reynolds numbers.

The validity of this model is also confirmed by data on interferometry of the flow and photoelectric recording of the oscillations of the integral arc radiation in a plasmotron channel [3].

The two-mode flow model here discussed was used in [4], to calculate the characteristics of an atmospheric-pressure plasmotron, neglecting radiation from the arc. Satisfactory agreement of the experimental and theoretical data was noted for helium arcs; this may indicate the correctness of the approach adopted.

In the present work, a two-mode model is developed for the case of an arc in which the radiation cannot be neglected. As before, a stabilized section in which the flow parameters and arc characteristics do not change along the channel axis is considered. In this case, all the heat liberated is transmitted to the channel wall on account of heat conduction and radiation. The atmospheric-pressure arc plasma in the channel of relatively small diameter is practically transparent to the intrinsic radiation, and therefore it may be taken into account in the approximation of an optically thin layer.

The flow in the channel is represented as two zones: an internal electrically conducting laminar zone and an external electrically nonconducting turbulent zone. Then the energy equation and boundary conditions for these zones are, respectively, as follows

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dS}{dr}\right)+\sigma E^2-\varepsilon=0,\ S(0)=S_0,\ S(r_\sigma)=S_\sigma; \tag{1}$$

$$\frac{1}{r}\frac{d}{dr}\left(rA_q\frac{dh}{dr}\right)=0,\ h(r_\sigma)=h_\sigma,\ h(R)=h_{\text{ст}}. \tag{2}$$

To simplify the calculation, no account is taken here of the intermediate zone in which the transition from laminar to turbulent flow occurs.

It is usual to solve Eq. (1) by linearizing the dependences of the electrical conduction and radiation on the thermal function S. The values of the thermal function at the boundary of the internal and external zones must be the same here for both approximations. However, this approach is not very accurate, since in reality the radiation becomes significant at markedly higher temperatures (and hence S) than the electrical conduction. In addition, the dependence $\sigma(S)$ for large S deviates strongly from a straight line, and therefore the linear approximation leads to great error. To eliminate these difficulties, the electrically conducting zone is divided into two layers: a central radiating layer and an annular layer in which there is no radiation. In each of these layers, the nonlinear dependences $\sigma(S)$ and $\varepsilon(S)$ are replaced by linear dependences. Then the energy equation for the first and second layers with the corresponding approximations is written in the form

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dS_{\text{I}}}{dr}\right)+\sigma_{\text{I}}E^2-\varepsilon=0,$$

$$S_{\text{I}}(0)=S_0,\ S_{\text{I}}(r_\varepsilon)=S_\varepsilon,\ S_\varepsilon\leqslant S_{\text{I}}\leqslant S_0. \tag{3}$$

Here

$$\sigma_{\text{I}}=b_{\text{I}}^2(S_{\text{I}}-S_\varepsilon)+\sigma_\varepsilon,\ \varepsilon=a^2(S_{\text{I}}-S_\varepsilon), \tag{4}$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dS_{\text{II}}}{dr}\right)+\sigma_{\text{II}}E^2=0, \tag{5}$$

where

$$S_{\text{II}}(r_\varepsilon)=S_\varepsilon,\ S_{\text{II}}(r_\sigma)=S_\sigma,\ S_\sigma\leqslant S_{\text{II}}\leqslant S_\varepsilon,$$

$$\sigma_{\text{II}}=b_{\text{II}}^2(S_{\text{II}}-S_\sigma). \tag{6}$$

It is clear that Eqs. (4) and (6) are a considerably better approximation of the actual dependences $\sigma(S)$ and $\varepsilon(S)$ than in the case of a single electrically conducting zone.

Solving Eqs. (3) and (5) gives the following expression for the thermal function

$$S_{\mathrm{I}} = S_\varepsilon - \frac{\sigma_\varepsilon E^2}{m^2} + \left(S_0 - S_\varepsilon + \frac{\sigma_\varepsilon E^2}{m^2}\right) J_0(mr), \tag{7}$$

$$S_{\mathrm{II}} = S_\sigma + (S_\varepsilon - S_\sigma)\frac{Y_0(x) J_0(x_\sigma) - Y_0(x_\sigma) J_0(x)}{Y_0(x_\varepsilon) J_0(x_\sigma) - Y_0(x_\sigma) J_0(x_\varepsilon)}, \tag{8}$$

where $m^2 = b_I E^2 - a^2$, $x = r b_{II} E$, $x_\sigma = r_\sigma b_{II} E$, $x_\varepsilon = r_\varepsilon b_{II} E$.

Solving Eq. (2) entails knowing the turbulent heat-transfer coefficient. The empirical formulas for $A_q$ in the literature are difficult to use in analytical calculations because of their complexity and unwieldiness. A simplifying approach to the determination of $A_q$ is considered below. The turbulent viscosity is found for flow in a cylindrical channel, on the basis of expressions for the tangential turbulent stress

$$\tau = \mu_t \frac{du}{dr}, \quad \tau = \tau_w \frac{r}{R}.$$

Hence taking into account that $\tau_w/\rho = V^2$, it follows that

$$\mu_t = \rho \frac{V^2}{R \dfrac{du}{dr}}. \tag{9}$$

As is known, a power-law (to power 1/7) velocity distribution over the cross section may be written for the flow in tubes at Re up to $10^5$

$$\frac{u}{u_m} = \left(1 - \frac{r}{R}\right)^{1/7}. \tag{10}$$

Since $\bar{u} = 0.817 u_m$ for the power 1/7, it follows that

$$u = 1{,}224\bar{u}\,(1 - \bar{r})^{1/7}. \tag{11}$$

Using the expression $V = \sqrt{(\eta/8)}\bar{u}$, where $\eta = 0.3164\sqrt[4]{\bar{u}d/\nu}$ (the Blasius resistance law) and substituting Eq. (11) into Eq. (9), it is found that

$$\mu_t = 0{,}0806 \sqrt[4]{\frac{\mu G^3}{R^3}}\,\bar{r}\,(1 - \bar{r})^{6/7}. \tag{12}$$

A similar approach was used in [5] for determining $\mu_t$; to simplify the analytical calculation, the experimental dependence of the kinematic turbulent viscosity was approximated by two parabolas. In fact, this operation is equivalent to approximating the actual velocity profile by an analytically simple expression.

Using the data of [6], which indicate that the turbulent Prandtl number ($Pr_t$) in the region of flow with a logarithmic profile (close to the power law adopted here) does not depend on Re, Pr or the distance from the wall, and is $\sim 0.87$, the final expression obtained for $A_q$ is

$$A_q = \frac{\mu_t}{\mathrm{Pr}_t} = 0{,}0926 \sqrt[4]{\frac{\mu G^3}{R^3}}\,\bar{r}\,(1 - \bar{r})^{6/7}. \tag{13}$$

Since the power-law velocity profile is not significantly altered by the nonisothermal character of the flow [7], Eq. (13) may also be used for the external gas flow blown through the arc, referring the gas properties (in particular, $\mu$) to its mean temperature.

The solution of Eq. (7) with $A_q$ from Eq. (13) takes the form

$$h = \frac{(h_\sigma - h_w) f(\bar{r}) + f(\bar{r}_\sigma) h_w - 0{,}082 h_\sigma}{f(r_\sigma) - 0{,}082}, \tag{14}$$

where

$$f(\bar{r}) = \frac{0{,}0187}{1 - (1 - \bar{r})^{1/7}} - 0{,}1231 \ln[1 - (1 - \bar{r})^{1/7}] +$$

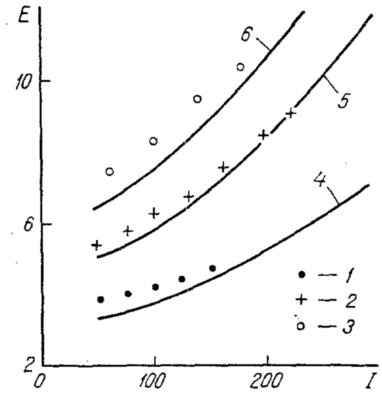


Fig. 1. E − I characteristics of an arc in a channel with an argon flow. Experiment: 1) [14]; 2) [12]; 3) [13]; 4-6) theoretical data: 1, 4) d = 15 mm, G = 1.1 g/sec; 2, 5) 8, 1; 3, 6) 10, 4. E, V/cm; I, A.

$$+ 0{,}1104 \ln [(1-\bar{r})^{2/7} + 1{,}802(1-\bar{r})^{1/7} + 1] + 0{,}1941 \operatorname{arctg}[1{,}279(1-\bar{r})^{1/7} - 0{,}797] + \tag{15}$$

$$+ 0{,}2411 \operatorname{arctg}[1{,}025(1-\bar{r})^{1/7} + 0{,}228] + 0{,}2238 \operatorname{arctg}[2{,}307(1- \\ -\bar{r})^{1/7} + 2{,}078] - 0{,}2209[(1-\bar{r})^{1/7} - 0{,}78]^2.$$

The use of matching conditions at the boundary between the layers

$$S_{\rm I}(r_\varepsilon) = S_{\rm II}(r_\varepsilon) = S_\varepsilon, \quad \left.\frac{dS_{\rm I}}{dr}\right|_{r=r_\varepsilon} = \left.\frac{dS_{\rm II}}{dr}\right|_{r=r_\varepsilon}$$

leads to a system of two equations with three unknowns $r_\sigma$, $r_\varepsilon$, E

$$J_0(mr_\varepsilon) = \frac{\sigma_\varepsilon E^2}{m^2(S_0 - S_\varepsilon) + \sigma_\varepsilon E^2}, \tag{16}$$

$$\frac{m^2(S_0 - S_\varepsilon) + \sigma_\varepsilon E^2}{mb_{\rm II}E(S_\varepsilon - S_\sigma)} J_1(mr_\varepsilon) = \frac{Y_0(x_\sigma)J_1(x_\varepsilon) - Y_1(x_\varepsilon)J_0(x_\sigma)}{Y_0(x_\varepsilon)J_0(x_\sigma) - Y_0(x_\sigma)J_0(x_\varepsilon)}. \tag{17}$$

To close the system in Eqs. (16) and (17), a third equation must be added. To this end, the heat fluxes at the boundary of the arc zone and the external turbulent flow are equated

$$\left.\frac{dS}{dr}\right|_{r=r_\sigma} = A_q \left.\frac{dh}{dr}\right|_{r=r_\sigma}.$$

This leads to the required equation

$$-0{,}0132 \sqrt[4]{\frac{\mu G^3}{R^3}} \frac{h_\sigma - h_{\rm W}}{\bar{r}_\sigma[f(\bar{r}_\sigma) - 0{,}082]} = b_{\rm II}E(S_\varepsilon - S_\sigma) \frac{Y_0(x_\sigma)J_1(x_\sigma) - Y_1(x_\sigma)J_0(x_\sigma)}{Y_0(x_\varepsilon)J_0(x_\sigma) - Y_0(x_\sigma)J_0(x_\varepsilon)}, \tag{18}$$

which, together with Eqs. (16) and (17), forms a closed transcendental system, allowing the unknowns $r_\sigma$, $r_\varepsilon$, E to be determined for specified channel radius, working-gas flow rate, and axial value of the thermal function.

The arc current is found using the Ohm's law in integral form

$$I = 2\pi E \int_0^R \sigma r dr.$$

After substituting in σ from Eqs. (4) and (6) and S from Eqs. (7) and (8) and integrating, it follows that

$$I = 2\pi E \left[ \frac{a^2\left(S_0 - S_\sigma + \dfrac{\sigma_\varepsilon E^2}{m^2}\right) J_1(mr_\varepsilon) r_\varepsilon}{mE^2} - \frac{S_\sigma}{E^2 \ln \dfrac{r_\sigma}{R}} - \frac{r_\varepsilon^2 \sigma_\varepsilon a^2}{2m^2} \right]. \tag{19}$$

On the basis of the formulas obtained, the basic characteristics of an electrical arc in a cylindrical channel with the injection of a turbulent argon flow are calculated. The values of the transfer coefficients required here are taken from [8-10]. The basis for the choice of these data was outlined in [11].

Some results of the calculation are shown in Fig. 1, in comparison with experimental data [12-14]. The experimental arc characteristics, like the calculation, apply only to the stabilized section. As follows from Fig. 1, the agreement of the theoretical and experimental E—I characteristics is satisfactory.

It may be concluded from the foregoing that the given two-mode model of flow in a plasmotron channel may be used at sufficiently large arc currents and moderate Reynolds numbers of the turbulent argon flow. The analytical expressions obtained may be used in calculations of the basic characteristics of the stabilized section of the arc in a plasmotron with corresponding operating conditions.

## NOTATION

$\sigma$, electrical conductivity; $\varepsilon$, density of the integral radiation; E, electric field strength; I, strength of arc current; h, enthalpy; $S = \int_0^T \lambda dT$, thermal function: $\tau$, tangential stress; $A_q$, turbulent heat-transfer coefficient; $\mu$, $\mu_t$, molecular and turbulent viscosity; $\nu$, kinematic viscosity; $\rho$, density; u, velocity; $u_m$, maximum velocity; $\bar{u}$, mean velocity; V, dynamic velocity; G, mass flow rate; $\bar{r} = r/R$, reduced radius; R, channel radius; $J_0$, $Y_0$, zero-order Bessel functions of the first and second kind; $J_1$, $Y_1$, first-order Bessel functions of the first and second kind. Indices: 0, value at the axis; w, channel wall; $\sigma$, boundary of the conducting zone; $\varepsilon$, boundary of the radiating zone.

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